jet entrains the dye and lowers its concentration. To obtain comparable data for different upstream flow conditions, a new parameter V^* was introduced and defined as the ratio of the jet velocity to the freestream velocity. Table 1 shows a summary of the experimental data collected, in which V^*_{min} is the minimum value of V^* that produced attached flow at the given flow Reynolds numbers.

Conclusions

We have conducted a flow visualization of the flow past a cylinder with and without a rear stagnation jet. It has been found that the massively separated wake behind the cylinder can be completely eliminated and the flow around the cylinder made to behave as inviscid flow by a rear stagnation jet with a minimum dimensionless parameter V^* of 25. Increasing V^* over this critical value showed no further noticeable changes on the flow pattern. The details of the flowfield are under a comprehensive numerical investigation by the authors.

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Multigrid Techniques for Hypersonic Viscous Flows

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Introduction

VISCOUS hypersonic flows are dominated by strong shock-wave/boundary-layer interactions that affect the features of the flowfield and may cause boundary-layer separation and laminar-turbulent transition. For an accurate prediction of these complex phenomena, the numerical solution of such flows requires very fine grids, with a loss in the computational efficiency.

In this context, the multigrid technique is a viable (and necessary) tool to accelerate the convergence of the calculation, by allowing the use of large time steps on coarse grids so that disturbances are rapidly expelled from the computational domain. However, up to now only a few applications of the technique to hypersonic flows have been reported.

Turkel et al.[1] have applied the standard full approximation storage (FAS) full multigrid (FMG) method[2,3] to compute viscous hypersonic flows over airfoils and blunt biconic bodies. They have concluded that the smoothing of coarse grid corrections is essential to ensure fast convergence. Leclercq and Stoufflet[4] have developed a characteristic multigrid method to solve inviscid hypersonic flows on unstructured and un-nested grids. Applications of the method to compute flows around airfoils and blunt bodies have shown that the use of an upwind prolongation operator and a geometrical restriction operator is the most efficient strategy for hypersonic flows. Koren and Hemker[5] have developed a damped direction-dependent multigrid method to solve steady Euler equations by means of point relaxation sweeps. They have shown that an upwind prolongation based on a MUSCL reconstruction of the fine grid solution, coupled with a locally damped classical residual restriction operator, is the most appropriate strategy for blunt body flows. Recently, Radespiel and Swanson[6] have investigated various multigrid schemes with semicoarsening to overcome stiffness problems due to the high cell aspect ratios, needed for high Reynolds number flows. They have used standard grid transfer operators coupled with local damping of the restriction operator and an upwind total variation diminishing (TVD) discretization, concluding that sequential semicoarsening gives the best results in terms of convergence rate, even though the computational work per multigrid cycle is about three times that of a full coarsening multigrid approach. Blazek et al.[7] have compared several implicit residual smoothing operators in combination with multigrid, showing that, for hypersonic flows, direction-dependent operators have better damping and convergence properties with respect to central ones.

In the present work, we have developed a multigrid technique for viscous hypersonic flows. The technique belongs to the class of the FAS-FMG methods, and it uses a "V-cycle" multigrid strategy and direction-dependent grid transfer operators. The full Navier-Stokes equations are solved by a finite volume approach with cell centered formulation, based on an adaptive dissipation scheme.[8] Time integration is performed by a multistage explicit Runge-Kutta algorithm coupled with a direction-dependent, implicit residual smoothing, to extend the stability region of time-stepping schemes.[3,8,9] The multigrid algorithm is based on a conservative residual restriction, geometrical solution restriction, and direction-dependent coarse grid correction prolongation.

Numerical Solution

The Navier-Stokes equations (in conservation form) are solved by means of a cell-centered finite volume formulation, whereby the computational domain is decomposed into arbitrary quadrilateral cells (i, j). Space and time discretization are separated by using the method of lines, thus reducing the governing equations to a system of ordinary differential equations for each computational cell. The basic numerical algorithm employs symmetric discretization of both the inviscid and viscous terms.[8]

Adaptive dissipation is added to prevent oscillations and even/odd point decoupling. On the finest grid the adaptive dissipation is a blending of first- and third-order derivatives,[3,9] with the shock sensor definition of Turkel et al.,[1] that ensures a TVD character of the scheme across shocks and improves the accuracy and the rate of convergence. A dissipation model based on first-order derivatives (computationally efficient) has been employed on coarse grids.[3,6] Time integration is performed by using a five-stage Runge-Kutta algorithm.

Implicit Residual Smoothing

To extend the stability bounds of the algorithm, we have analyzed the use of two direction-dependent implicit residual smoothing operators. Let $\boldsymbol{R}_{i,j}^{(k)}$ be the residual of the (discretized) equations at the kth stage of the Runge-Kutta algorithm.

In the first approach, acoustic residual smoothing (ARS), the residual is smoothed in the two directions according to the following algorithm

$$-\epsilon_i b_i \bar{\boldsymbol{R}}_{i-1,j}^{(k)} + [1+\epsilon_i(a_i+b_i)]\bar{\boldsymbol{R}}_{i,j}^{(k)} - \epsilon_i a_i \bar{\boldsymbol{R}}_{i+1,j}^{(k)} = \boldsymbol{R}_{i,j}^{(k)} \quad (1)$$

$$-\epsilon_j b_j \bar{\bar{\boldsymbol{R}}}_{i,j}^{(k)} + [1+\epsilon_i(a_i+b_i)]\bar{\bar{\boldsymbol{R}}}_{i,j-1}^{(k)} - \epsilon_j a_j \bar{\bar{\boldsymbol{R}}}_{i,j+1}^{(k)} = \bar{\boldsymbol{R}}_{i,j}^{(k)} \quad (2)$$

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The coefficients a_i, b_i (a_j, b_j) are assumed to depend on the directions of propagation of pressure disturbances, according to the local normal Mach number.

In the second method, eigendirection residual smoothing (ERS), a characteristic decomposition of the equations is introduced, and the residual is smoothed in the two directions according to the following algorithm:

$$Q_{i,j} = LR^{(k)}_{i,j} \tag{3}$$

$$-\epsilon_i b_i^l \tilde{Q}^l_{i-1,j} + [1 + \epsilon_i(a_i^l + b_i^l)]\tilde{Q}^l_{i,j} - \epsilon_i a_i^l \tilde{Q}^l_{i+1,j} = Q^l_{i,j} \tag{4}$$

$$-\epsilon_j b_j^l \tilde{\tilde{Q}}{}^l_{i,j-1} + [1 + \epsilon_j(a_j^l + b_j^l)]\tilde{\tilde{Q}}{}^l_{i,j} - \epsilon_j a_j^l \tilde{\tilde{Q}}{}^l_{i,j+1} = \tilde{Q}^l_{i,j} \tag{5}$$

$$\tilde{\tilde{R}}{}^{(k)}_{i,j} = L^{-1}\tilde{\tilde{Q}}_{i,j} \tag{6}$$

where the coefficients a_i^l, b_i^l(a_j^l, b_j^l) depend on the inviscid eigenvalues $\lambda_i^l(\lambda_j^l)$, and L and L^{-1} are the left and right eigenvector matrices of the inviscid normal flux Jacobians. The smoothing coefficients ϵ_i, ϵ_j depend on the Courant number and grid characteristics.[8]

Multigrid Strategy

The multigrid technique has been shown to be a viable tool for convergence acceleration for transonic and supersonic flow simulations, by allowing the use of large time steps on coarse grids, thus acting as a low pass filter. For hypersonic flows the grid transfer operators play a crucial role to ensure fast convergence and stable solutions.

In the present work we have developed a multigrid technique based on the FAS-FMG[2,3] that uses a V-cycle strategy, where on each multigrid level multiple Runge-Kutta iterations are performed on the coarse grids. The solution is transferred from each mesh to the next coarser one after these iterations, by means of a volume weighted-average restriction operator and a conservative residual restriction. The coarse grid solution evolves by performing two or more Runge-Kutta iterations. This process is repeated until the coarsest grid is reached. The solution is then interpolated back to the finest mesh by a coarse grid correction prolongation operation without any intermediate calculation. The solution vector is initialized on the coarser grid (m) of the basic sequence of grids and iterated a fixed number of cycles using the FAS scheme. Then, the solution is interpolated to the next finer multigrid level ($m + 1$), and this process is repeated until the finest grid of the sequence has been reached.

To inhibit upstream propagation of high-frequency disturbances on finer meshes, we have introduced a direction-dependent prolongation operator, according to the direction of propagation of acoustic disturbances [acoustic correction prolongation (ACP)]. Let h and $2h$ indicate two successive grid levels (fine and coarse), then prolongation operation yields

$$W_h = W_h + P^h_{2h}\Delta W_{2h} \tag{7}$$

where P^h_{2h} is the prolongation operator defined as

$$P^h_{2h} = \{[I - \epsilon(a_j\Delta_+ - b_j\Delta_-)][I - \epsilon(a_i\Delta_+ - b_i\Delta_-)]\}_h B^h_{2h} \tag{8}$$

and the a and b depend on the direction of propagation of pressure disturbances, ϵ is a constant value, and B^h_{2h} is a bilinear interpolation operator. Moreover, a prolongation operator based on characteristic decomposition [eigendirection correction prolongation (ECP)] has also been employed.

Stability Analysis

A one-dimensional linear advection equation has been analyzed to evaluate the stability properties, the damping behavior of the solution strategy, and the effects of direction-dependent operators.[8] The amplification factor for unsmoothed, central smoothed, and direction-dependent smoothed schemes are shown in Fig. 1, together with the amplification factor of the complete multigrid algorithm (which uses direction-dependent prolongation operator). As the figure shows, the damping behavior of the direction-dependent smoothed scheme is better than that of the unsmoothed and central smoothed ones, both at high and low frequencies. Moreover, the damping properties of the multigrid strategy in reducing the errors over the "entire" frequency spectrum is clearly shown.

Results

Laminar hypersonic flows have been computed to show the influence of grid transfer operators and flow complexity on multigrid. The first test case corresponds to the flow over a flat plate (whose length is $L_p = 35.0$ cm) at $M_\infty = 5.0$; $Re/m = 6 \cdot 10^6$; $T_\infty = 80$ K and $T_{\text{wall}} = 288$ K. The grid used has 176×48 cells, with a maximum aspect ratio of about 30, corresponding to a normal nondimensional spacing at the wall $\Delta y_{\min} = 2 \cdot 10^{-4}$. All computations have been performed with a Courant number equal to the optimum value ($\sigma = 2.5$), and the effects of the different smoothing and grid transfer operators have been assessed. In particular, Fig. 2 shows the convergence rates corresponding to the multigrid strategy based on the use of direction-dependent smoothing (ERS) coupled with the standard correction prolongation (SCP) and direction-dependent prolongation operators (ECP and ACP). The single grid-acoustic residual smoothing (SG-ARS) and the multigrid solution with standard operators [standard residual smoothing (SRS) and SCP] are also reported. The advantages of using direction-dependent prolongation are clearly evident: for the same convergence level a gain of about 30% in total CPU time is obtained. Moreover, using eigendirection decomposition prolongation yields a slightly better convergence than ACP,

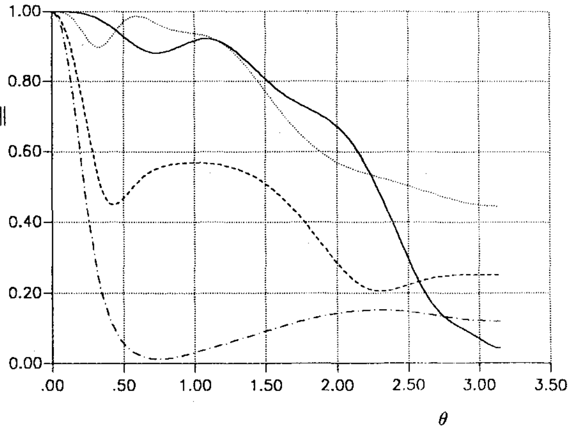

Fig. 1 Amplification factor vs phase angle: —, SG unsmoothed; · · ·, SG SRS; ---, SG ARS; —·—, MG ARS/ACP.

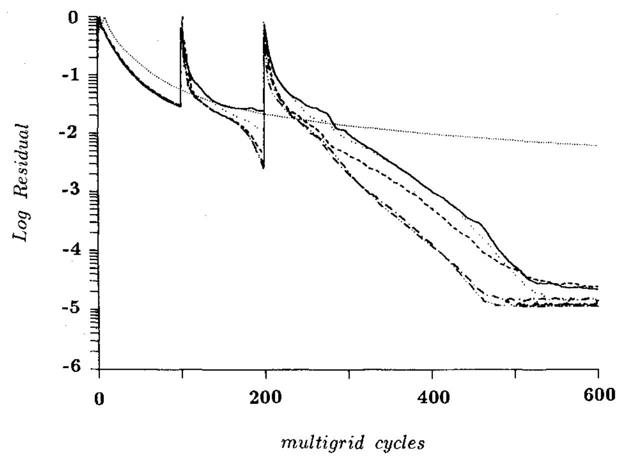

Fig. 2 Hypersonic boundary layer: influence of grid transfer operators on convergence: · · ·, SG ARS; —, MG SRS SCP; ---, MG ARS/ACP; ·· ··, MG ERS/SCP; —·—, MG ERS/ACP; -··-, MG ERS/ECP.

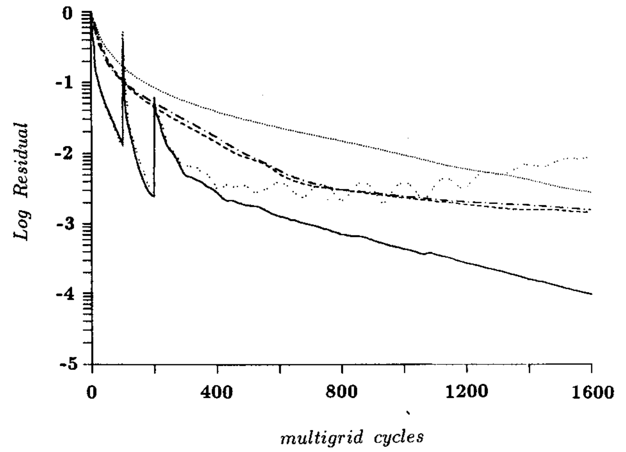

Fig. 3 Strong shock-wave/boundary-layer interaction, influence of smoothing and grid transfer operators on convergence: · · ·, SG unsmoothed; ---, SG SRS; —·—, SG ERS; ·· ·· MG SRS/SCP; —, MG ERS/ECP.

even though the computational overhead per cycle is greater (roughly 45% larger).

The second test case corresponds to the hypersonic (laminar) flow over a compression ramp. This test case has been selected to show the influence of both grid transfer operators and flow complexity on the convergence rate. The flow conditions are those of the previous case and are such that an extended separation region is present due to the strong viscous-inviscid interaction. The geometry is composed of a flat plate of length $L_p = 25$ cm, followed by a 15-deg wedge of length $L_w = 10$ cm. Several computations have been performed, and the convergence rates are shown in Fig. 3. In particular, the convergence of the single grid solution with and without smoothing is compared vs the multigrid with standard grid transfer operators, as well as direction-dependent operators. The results show that in the presence of a large separation extent convergence is mainly driven by the development of the recirculation region. Moreover, the standard multigrid strategy does not converge.

Conclusions

In the present work a multigrid technique for viscous hypersonic flows has been developed. The technique is based on the FAS-FMG method and it uses a V-cycle strategy. Applications to hypersonic flows show that the grid transfer operators play a crucial role for the robustness and effectiveness of the technique: 1) to inhibit upstream propagation of high-frequency disturbances, the prolongation operator must have a directionality character; 2) the robustness of the algorithm is increased by direction-dependent implicit residual smoothing; and 3) in the presence of strong shock-wave boundary-layer interaction the convergence is affected by the development of the recirculation region.

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Boundary Formulations for Sensitivity Analysis Without Matrix Derivatives

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Introduction

A NEW hybrid approach to continuum structural shape sensitivity analysis employing boundary element analysis (BEA) is described that uses iterative reanalysis to obviate the need to factor perturbed matrices in the determination of surface displacement and traction sensitivities via a univariate perturbation/finite difference (UPFD) step. Sensitivities of derived surface stresses and interior displacements and stresses are determined by an analytical formulation using these surface sensitivities. This algorithm is shown to be superior to a related semi-analytical approach. Test cases are discussed, and it is concluded that the technique is viable for shape sensitivity analysis from the software engineering perspective because it avoids computation of derivatives of BEA matrices.

Shape design sensitivity analysis (DSA) denotes computing rates of change of an object's response with respect to changes in design variables that control its shape. Implicit differentiation[1,2] is effective for computing sensitivities of surface displacement and tractions and avoids factoring perturbed matrices. However, it requires derivatives of BEA coefficient matrices. For a general BEA program, the formulation and implementation of this capability is a considerable effort. For sophisticated elements (i.e., trimmed patch elements),[3,4] such activity necessitates computation of sensitivities of element differential geometry, which requires sensitivities of the surface geometric model. The boundary integral equations (BIEs) associated with implicit differentiation are also more complex than those in standard BEA, thus requiring more time in the numerical integration process.

This new technique obviates the need to compute BEA matrix sensitivities in the DSA process by employing a UPFD strategy to determine surface displacement and traction sensitivities in concert with an iterative reanalysis technique.[5] The effect of this strategy is two-fold. The UPFD approach allows the immediate reuse of existing subroutines for computation of BEA matrix coefficients in the DSA process. The reanalysis technique provides for economical response computation of univariately perturbed models without factoring perturbed matrices. A formulation is shown, contrasting this new approach with a semi-analytical technique for DSA presented elsewhere.[6] Examples show that this new approach is equivalent to the more conventional analytic sensitivity analyses and

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